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Comparison of Holt-Winters Exponential Smoothing (HWES) and Singular Spectrum Analysis (SSA) Methods on Forecasting the Number of PT KAI Passengers in Indonesia

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Abstract

The study examines the application of two forecasting methods, Holt Winters Exponential Smoothing (HWES) and Singular Spectrum Analysis (SSA), in forecasting the number of passengers at PT Kereta Api Indonesia. The result show that the application of HWES with the additive model produces optimal smoothing parameter with $(\boldsymbol{\alpha}) = \mathbf{0.4}$, *beta* $(\boldsymbol{\beta}) = \mathbf{0,2}$ *and gamma* $(\boldsymbol{\gamma}) = \mathbf{0.7}$, *with a MAPE value of 10.75%. Meanwhile, the HWES multiplicative model yields smoothing parameters of alpha* $(\boldsymbol{\alpha}) = \mathbf{0.6}$, *beta* $(\boldsymbol{\beta}) = \mathbf{0,1}$ *and gamma* $(\boldsymbol{\gamma}) = \mathbf{0.8}$, *resulting in a MAPE value 14.50%. The SSA method with a Window Length* $(\boldsymbol{L}) = \mathbf{5}$ *produces a MAPE value of 13.33%. The comparison of MAPE values between the HWES additive is superior with a MAPE of 10.75%. The forecasting of the number of passangers for PT Kereta Api Indonesia using the best method, Holt Winters Exponential Smoothing Additive, for the period from January to December 2024, shows the lowest number of passanger in Agust and the highest in Januari.*

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INTRODUCTION

Land transportation such as trains is a widely used public transportation mode due to its numerous advantages and the positive reception it has garnered from the public [6]. Train transportation encompasses a comprehensive system that includes infrastructure, facilities, workforce, as well as criteria, requirements, and measures for transportation management [15]. PT Kereta Api Indonesia (KAI) is a state-owned enterprise (SOE) focusing on transportation services, providing facilities and infrastructure to support the smooth operation of passenger and freight transportation. The objective of PT Kereta Api Indonesia is to implement and support the government's economic policy system through the provision of service-based transportation. Additionally, the company aims to generate profits by delivering transport services that enhance service quality, such as stations and other facilities for cargo loading and unloading processes [12].

Policies designed to enhance national railway services aim, among other goals, to strengthen the role of trains as a mass transportation mode in urban development areas and as an intercity transportation means connecting national activity centers. Additionally, these policies aim to facilitate easy access to ports and airports, thereby supporting the efficiency of freight shipping and the organization of national logistics systems [9].

Figure 1. Graph of Indonesian Train Passengers 2014-2023
Source: Central Statistics Agency (BPS), 2023

Based on data from BPS shown in Figure 1, the number of passengers of PT Kereta Api Indonesia during the period 2014 to 2023 indicates a fluctuating pattern. The number of passengers using train services increased consecutively each year from 2014 to 2019, with the highest increase occurring in 2019, reaching 428,006 thousand passengers. However, the number of passengers declined in 2020 to 186,125 thousand passengers, with the lowest recorded in 2021 at 149,763 thousand passengers. This decline was due to not yet returning to pre-pandemic levels. Nevertheless, after the decrease in COVID-19 cases, the number of passengers using train services showed an increase in the following year. In 2023, the number of passengers using train services increased by 1.48%, reaching 371,538 thousand passengers. The high enthusiasm for train transportation can be observed from the historical annual increases shown in Figure 1, except during the pandemic period. The implementation of the national railway system must have measurable targets in the form of quantitative data, which can serve as tools to improve performance and ensure the success of the national railway system [10].

Considering the existing challenges, estimation or prediction is necessary as a decision-making tool to provide an overview of the number of passengers using train services in Indonesia. This approach is implemented by applying the Singular Spectrum Analysis (SSA) and Holt-Winters Exponential Smoothing (HWES) methods. SSA is an innovative and advanced time series analysis that combines concepts from classical time series analysis, multivariate statistics, multivariate geometry, dynamic systems, and signal processing [8]. This approach eliminates the need to test assumptions such as independence and residual normality, making it more suitable for both stationary and non-stationary data. The evaluation of the best model can be assessed using the Mean Absolute Percentage Error (MAPE) [1].

In addition to SSA, Holt-Winters Exponential Smoothing (HWES) is a time series forecasting method capable of addressing seasonal or trend characteristics based on data from previous periods [13]. The Holt-Winters Exponential Smoothing (HWES) method is based on three elements in the smoothing equations: level, trend, and seasonality [3]. This method is a technique applied to address issues arising from seasonality and/or trends in a time series, resulting from the combination of Holt and Winters' methods. Forecasting using the Holt-Winters Exponential Smoothing method is generally not strictly tied to time series principles such as autocorrelation significance and stationarity. However, when data exhibit seasonal patterns, systematic errors are often observed [2]. The predictions in this study are conducted to enhance existing operational efficiency and ensure that train passenger services in Indonesia can optimally meet demand.

Several previous studies have been conducted, including one by Fitri and Rahmat (2021), which discussed the comparison of SSA and SARIMA in forecasting rainfall in West Sumatra [7]. Another study by Safitri et al. (2017), titled "Comparison of Forecasting Using Exponential Smoothing Holt-Winters and ARIMA Methods," also explored similar forecasting methodologies [13]. Furthermore, Atoyebi et al. (2023) examined forecasting currency in circulation in Nigeria using the Holt-Winters Exponential Smoothing Method [4]. Additionally, findings from a study by Nurvianti et al. (2019) explored the "Comparison of Forecasting the Number of Train Passengers in DKI Jakarta Using the Double Exponential Smoothing and Triple Exponential Smoothing Methods" [11]. Based on these studies, this research aims to examine "Comparison of Holt-Winters Exponential Smoothing (HWES) and Singular Spectrum Analysis (SSA) Methods in Forecasting the Number of Passengers of PT KAI in Indonesia." The data used consists of train passenger statistics obtained from the Indonesian Central Bureau of Statistics (BPS) for the period between January 2014 and December 2023[5].

METHOD

Data source

The data used in this study is secondary data on the number of passengers of PT Kereta Api Indonesia (KAI) for the period from January 2014 to December 2023. A total of 120 data points were obtained from the official website of the Central Bureau of Statistics (BPS) at www.bps.go.id.

Research Variables and Data Structure

The variables used in this study are the total passenger data of PT Kereta Api Indonesia, including passengers from the Jabodetabek region, non-Jabodetabek areas, and Sumatra, from January 2014 to December 2023. A total of 120 data points were divided into two parts: training and testing data. The training data, comprising 80% of the total (96 data points), was used to determine the model, while the testing data, comprising 20% of the total (24 data points), was used for forecasting. The structure of the data used in this study is presented in Table 1 as follows:

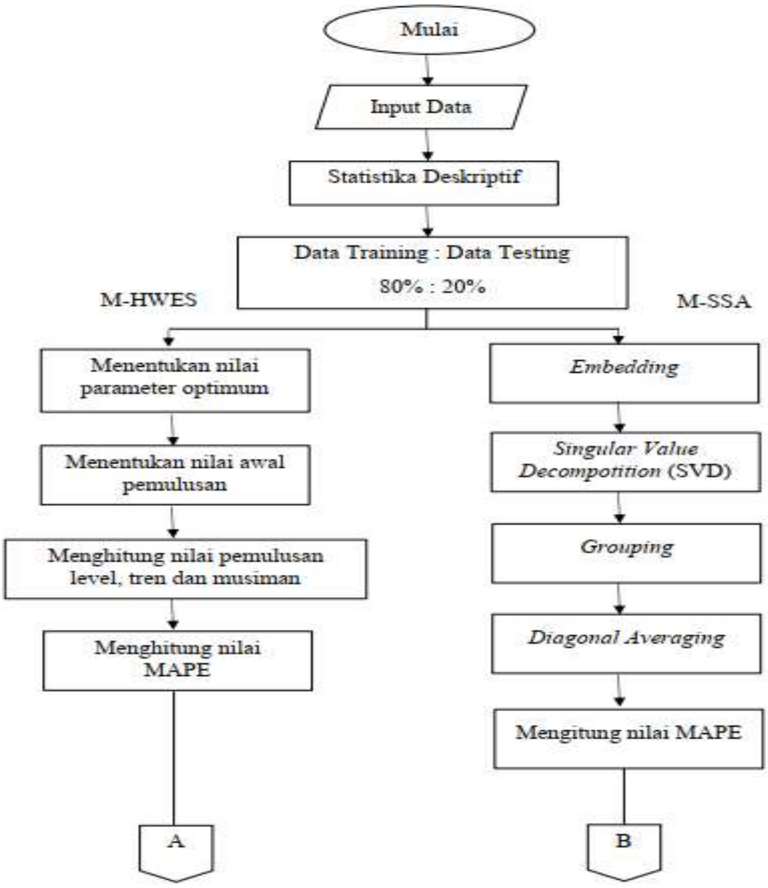
Table 1. Struktur Data

No	Waktu (t)	X_t

1	Januari 2014	X_1
2	Februari 2014	X_2
3	Maret 2014	X_3
4	April 2014	X_4
⋮	⋮	⋮
116	Agustus 2023	X_{116}
117	September 2023	X_{117}
118	Oktober 2023	X_{118}
119	November 2023	X_{119}
120	Desember 2023	X_{120}

Research Stages

The steps involved in forecasting the number of passengers of PT KAI are shown in the flowchart in Figure 2:



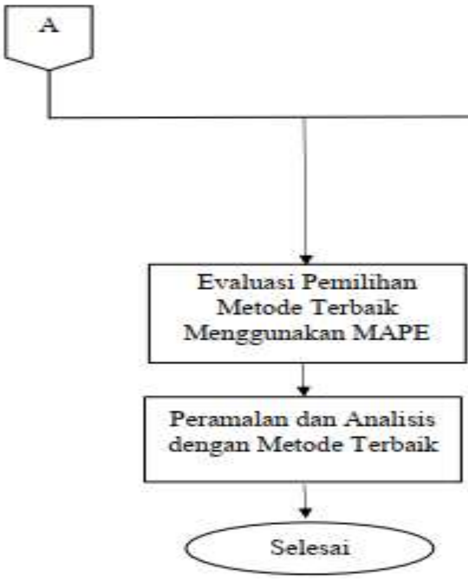


Figure 2. *Flowchart* Penelitian

Information:
M-HWES : Metode *Holt Winters Exponential Smoothing*
M-SSA : Metode *Singular Spectrum Analysis*

RESULTS AND DISCUSSION

Holt Winters Exponential Smoothing (HWES)

1. Optimum HWES Additive and Multiplicative Parameter Values

The Holt-Winters Exponential Smoothing (HWES) method is used to predict data with trend and seasonal patterns. This method is divided into two models: additive and multiplicative. In this study, decomposition analysis was conducted using Rstudio software to determine whether the data is more suited to the additive or multiplicative model. The results of this decomposition analysis were then used to select the appropriate model, as shown in Figure 3

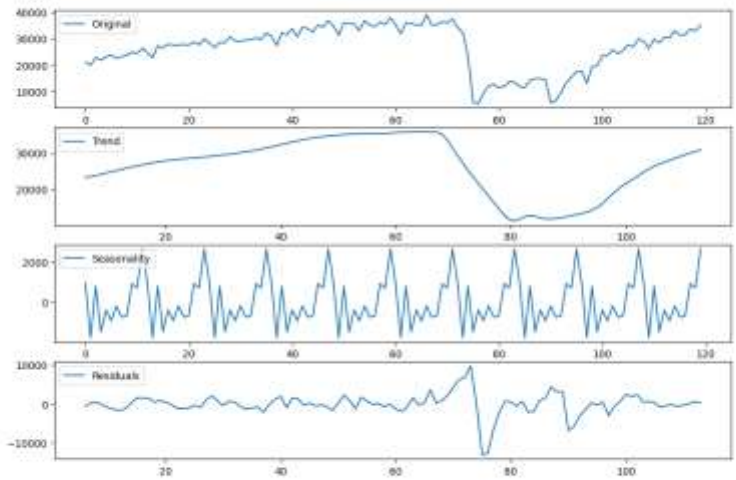


Figure 3. Data Visualization Graph

Based on Figure 3, it is observed that the data shows an increasing trend until 2018, followed by a decline from 2019 to 2021. The figure also indicates that the number of train passengers exhibits an annual seasonal pattern characterized by similar fluctuations, leading to the use of the additive model. In selecting the model, performance evaluation of the additive and multiplicative models was also conducted using Rstudio software, as presented in Table 2

Table 2. Performance Evaluation of HWES Models

Model HWES	**MAPE**
Training Set *Additive*	9.428754
Training Set *Multiplicative*	11.04344

The additive model demonstrates better performance due to its lower MAPE value, indicating smaller absolute errors and more accurate forecasting, as shown in Table 2. The analysis of the Holt-Winters Exponential Smoothing additive and multiplicative methods begins with determining the parameter values for alpha (α), beta (β), and gamma (γ). These parameters represent smoothing coefficients for level, trend, and seasonal components in the forecasting process. These values are determined based on training data with the help of Rstudio software. Based on parameter testing, the additive model achieved the smallest MAPE value of 36.50118. The results indicate that the parameter values are alpha (α) = 0.4, beta (β) = 0.2, and gamma (γ) = 0.7. Meanwhile, for the multiplicative model, the smallest MAPE value was 36.9038, with parameter values of alpha (α) = 0.6, beta (β) = 0.1, and gamma (γ) = 0.8.

1. Determining Initial Smoothing Values for HWES Additive and Multiplicative

In the implemented Holt-Winters model, the parameters alpha (α), beta (β), and gamma (γ) have been determined to optimize the forecasting of train passenger numbers in Indonesia. These parameters regulate the model's response to changes in level, trend, and seasonality. By utilizing these optimal parameters, both models can provide more accurate forecasts based on the characteristics and seasonal patterns of train passenger data in Indonesia. These initial smoothing values can be determined with the help of Rstudio software. Based on the Rstudio output, the initial smoothing values obtained are presented in Table 3:

Table 3. HWES Initial Smoothing Values

Model	*Additive*	*Multiplicative*
Nilai awal pemulusan level	31201	28357.9
Nilai awal pemulusan tren	-168.8	-116.37
Nilai awal pemulusan musiman		
S1	3756.45	1.10266
S2	-310.86	0.91725
S3	1815.54	1.04529
S4	-1693.6	0.79611
S5	-1526.7	0.78031
S6	-2918.7	0.82417
S7	-2416.4	0.81547
S8	-3754.8	0.84938

S9	-2313.2	0.98545
S10	783.689	1.14511
S11	1492.53	1.17158
S12	4343.08	1.23518

1. Calculating Level, Trend and Seasonal Smoothing Values in Additive and Multiplicative HWES

After obtaining the initial smoothing values for the Holt-Winters Exponential Smoothing additive and multiplicative methods, the next step is to calculate the smoothing values for level, trend, and seasonality using the following equations:

A. Level Smoothing Value

The level smoothing value for the additive model will be calculated using the following formula, with the smoothing parameter α=0.4α=0.4:

$$L_t = \alpha(X_t - S_{t-s}) + (1-\alpha)(L_{t-1} + b_{t-1})$$

$$L_{13} = 0.4(24676 - 4343.08) + (1-0.4)(31200.96 + (-168.8))$$

$$= 26752.46702$$

$$L_{14} = 0.4(22790 — (-150.6)) + (1-0.4) \times (26752.4 + (-1024.74))$$

$$= 24612.8767$$

$$\vdots$$

$$L_{120} = 0.4(35059 - 8071.037) + (1-0.4) \times (25027.249 + 541.285)$$

$$= 26136.30613$$

The level smoothing value for the multiplicative model will be calculated using the following formula, with the smoothing parameter α = 0.6:

$$L_t = \alpha \frac{X_t}{S_{t-s}} + (1-\alpha)(L_{t-1} + b_{t-1})$$

$$L_{13} = 0.6(\frac{24676}{1.2351}) + (1-0.6)(28357.906 + (-116.37))$$

$$= 23283.21048$$

$$L_{14} = 0.6(\frac{22790}{1.0948}) + (1-0.6)(23283.210 + (-612.203))$$

$$=21557.3114$$

$$\vdots$$

$$L_{120} = 0.6(\frac{35059}{0.973}) + (1-0.6)(33891.24 + 560.888)$$

$$= 35381.43228$$

B. Trend Smoothing Value

The trend smoothing value for the additive model can be calculated using the following formula, with the smoothing parameter β = 0.2

$$b_t = \beta(L_t - L_{t-1}) + (1-\beta)b_{t-1}$$

$$b_{13} = 0.2(26752.46702 - 31200.9672) + (1-0.2)(-168.8009)$$

$$= -1024.740756$$

$$b_{14} = 0.2(24612.87 - 26752.467) + (1-0.2) \times (-1024.74)$$

$= -1247.71067$

$\vdots$

$b_{120} = (26136.30613 - 25027.24938) + (1 - 0.2)(541.2858989)$

$= 654.8400687$

Berikut ini nilai pemulusan tren untuk model *multiplicative* akan dihitung menggunakan rumus berikut, dengan menggunakan nilai $(\beta) = 0.1$.

$b_t \ = \beta(L_t - L_{t-1}) + (1 - \beta)b_{t-1}$

$b_{13} = 0.1(23283.21 - 28357.906) + (1 - 0.1)(-1116.370)$

$= -612.203$

$b_{14} = 0.1(21557.311 - 23283.21) + (1 - 0.1) - 612.203$

$= -723.572$

$\vdots$

$b_{120} = 0.1(35381.432 - 33891.249) + (1 - 0.1)560.88$

$= 653.818$

C. Seasonal Smoothing Value

The seasonal smoothing value for the additive model can be calculated using the following formula, with the smoothing parameter

γ=0.7:

$S_t \ = \gamma(X_t - L_t) + (1 - \gamma)S_{t-1}$

$S_{13} = 0.7(24676 - 26752.46702) + (1 - 0.7)(4343.0819)$

$= \ -150.602344$

$S_{14} = 0.7(22790 - 24612.8767) + (1 - 0.7)(-150.602344)$

$= -1321.19439$

$\vdots$

$S_{120} = 0.7(35059 - 2613.30613) + (1 - 0.7)(8071.037594)$

$= 8667.196987$

he seasonal smoothing value for the multiplicative model can be calculated using the following formula, with the smoothing parameter γ=0.8

$S_t \ = \gamma \frac{X_t}{L_t} + (1 - \gamma)S_{t-s}$

$S_{13} = 0.8(\frac{24676}{23283.21}) + (1 - 0.8) \times 1.235$

$= 1.094$

$S_{14} = 0.8\left(\frac{22790}{21557.31}\right) + (1 - 0.8) \times 1.094$

$= 1.064$

$\vdots$

$S_{20} = 0.8\left(\frac{35095}{35381.43}\right) + (1 - 0.8) \times 0.973$

$= 0.987$

After calculating the smoothing values for level, trend, and seasonality in the Holt-Winters additive and multiplicative models, the next step is to calculate the Mean Absolute Percentage Error (MAPE). The formula for MAPE is as follows:

1. Forecast Accuracy of HWES Method

After obtaining forecasts using the additive and multiplicative models, the next step is to test their performance by comparing the forecasted values with the actual values in the test dataset. In this case, Mean Absolute Percentage Error (MAPE) is used as the evaluation metric. The formula for MAPE is:

A. MAPE ***Additive***

$$\text{MAPE} = \frac{1}{N}\sum_{t=1}^{N}\frac{|X_t-\hat{X}_t|}{X_t}\times 100\%$$

$$\text{MAPE} = \frac{1}{12}\sum_{12}^{1}\left(\frac{|29017-34788.62|}{20917}+\frac{|26259-30552.5|}{26259}+\cdots+\frac{|35059-33518.44|}{35059}\right)\times 100\%$$

$$\text{MAPE} = \mathbf{10.75}\%$$

B. MAPE ***Multiplicative***

$$\text{MAPE} = \frac{1}{N}\sum_{t=1}^{N}\frac{|X_t-\hat{X}_t|}{X_t}\times 100\%$$

$$\text{MAPE} = \frac{1}{12}\sum_{12}^{1}\left(\frac{|29017-31140.8|}{20917}+\frac{|26259-25797.6|}{26259}+\cdots+\frac{|35059-33302.2|}{35059}\right)\times 100\%$$

$$\text{MAPE} = \mathbf{14.50}\%$$

Based on the MAPE calculations, the results show a MAPE value of 10.75% for the additive model and 14.50% for the multiplicative model. A forecast is considered good if the MAPE value falls between 10% and 20%. Thus, MAPE values of 10.75% and 14.50% indicate that both models demonstrate good forecasting capabilities. However, the best MAPE value in the Holt-Winters Exponential Smoothing method is achieved using the additive model, making it the most suitable for forecasting the number of train passengers in Indonesia due to its smaller MAPE value.

***Singular Spectrum Analysis* (SSA)**

1. Decomposition

A. *Embedding*

The first step in the decomposition stage is the **embedding phase**. In this phase, the initial one-dimensional time series is transformed into a multidimensional series, referred to as the **trajectory matrix**. The **Window Length (L)** is a key parameter used in the decomposition process. In this case, with a total of 120 data points, the value of LL ranges between 2 and 48.

To simplify the process of finding the optimal LL, experiments were conducted with L=10,20,30,L=10,20,30, and 4040. The value of LL that produces the **minimum MAPE** (Mean Absolute Percentage Error) is then selected. The results are as follows:

Tabel 4. Result MAPE *Window Length* 1

***Window Length* (*L*)**	**10**	20	30	40
MAPE	**12,09**	33,58	30,44	26,51

The Window Length (LL) with the minimum MAPE of **12.09%** was identified. Using the same method, a more detailed search was conducted around L=10L=10 to find the most accurate Window Length (LL). The results are as follows.

Tabel 5. Hasil MAPE *Window Length* 2

***Window Length* (L)**	**5**	6	7	8
MAPE	**13.33334**	35.6326	41.84301	82.45932

The Window Length (LL) with a minimum MAPE of **13.33%** was obtained. With this MAPE value, it is expected that the prediction results will not deviate significantly from the actual data values. The chosen LL value is **L=5L=5** based on trial and error results. With the determined LL, the value of K=92K=92 was calculated to form a matrix with dimensions L×KL×K. Thus, the trajectory matrix (XX) constructed from the time series can be expressed as follows:

$$X = [X1: \dots : X92]$$
$$= (x)^{5,92}_{i,j}$$
$$= U_{5\times 5}$$
$$= \begin{bmatrix} 21092 & 19998 & 22836 & \cdots & 24676 \\ & \vdots & & \ddots & \vdots \\ 13722 & 13515 & 11901 & \cdots & 15317 \end{bmatrix}$$

Based on the trajectory matrix (XX), it has dimensions of 5×925×92, consisting of 5 rows and 92 columns with elements xijxij, where ii represents the row index and jj represents the column index. The transformed or decomposed matrix U(5×5)U(5×5) has dimensions of 5×55×5, providing a more compact representation of the original data for further analysis or pattern recognition within the data. This decomposition aims to derive a simpler or more meaningful representation of the original matrix.

B. *Singular Value Decompotition* (SVD)

The Singular Value Decomposition (SVD) process at this stage produces eigenvalues, eigenvectors, and principal components that assist in grouping the number of PT KAI passengers in Indonesia into several main components: trend, seasonality, and noise. The trajectory matrix XX is decomposed during the SVD step into 5 eigentriples, consisting of 5 eigenvalues, 5 eigenvectors, and 0 factor vectors

a. Eigen Value ($\sqrt{\lambda i}$)

The eigenvalues are obtained from the trajectory matrix, defined as S=XXTS=XXT

Tabel 6. Hasil *Eigen Value*

L	***Eigen Value* (S)**
1	363049150694
2	2324331344
3	693146756
4	306250960
5	192910323

b. The largest eigenvalue is **363,049,150,694**, which identifies the main component of the data, while the subsequent eigenvalues represent additional contributions to finer or more detailed variations in the data.

Eigen Vector (U_i)

The next step is to obtain the eigenvectors. This is done to facilitate the identification of data characteristics from the trajectory matrix.

$$U_{5\times 5} = \begin{bmatrix} 0.4479332 & 0.585970080 & -0.5165011 & \cdots & 0.1918556 \\ & \vdots & & \ddots & \vdots \\ -0.4446360 & -0.598734060 & -0.5078302 & \cdots & 0.2003549 \end{bmatrix}$$

c. *Principal Component* (V_i)

Nilai *principal component* dapat ditulis sebagai berikut:

$$V_{5\times 92} = \begin{bmatrix} 0.080761 & -0.0412505 & 0.000544 & \cdots & 0.1159830 \\ & \vdots & & \ddots & \vdots \\ 0.046038 & -0.181431 & 0.038864 & \cdots & 0.0230695 \end{bmatrix}$$

2. Reconstruction

Grouping

The initial step in the reconstruction phase is to perform a gradual grouping of eigentriples related to trend, seasonality, and noise. The **Grouping Effect (r)** is a parameter used in this grouping process. The grouping effect (rr) serves to limit the number of eigentriples that will be utilized in identifying the trend and seasonal components

a. Grouping Noise Components

The value of the **Grouping Effect (r)** parameter is determined based on the number of eigentriples that do not represent noise, as observed in the eigenvalue plot. Eigenvalues that decrease gradually in the plot indicate the presence of noise components

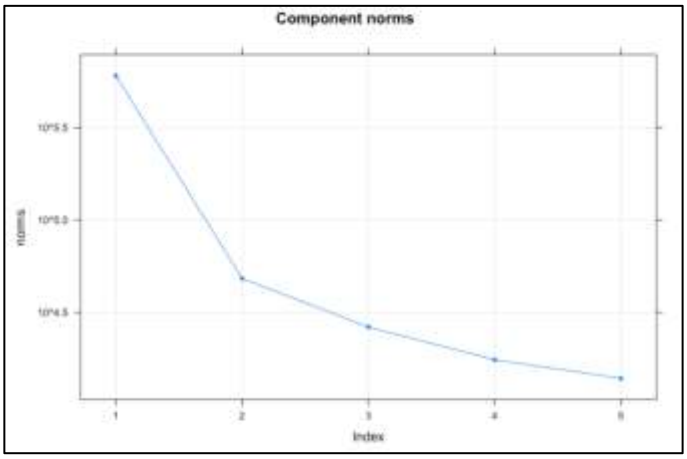


Figure 4. Plot of 5 Eigenvalues

Based on Figure 4, the eigenvalues begin to gradually decline from eigentriple 2 to eigentriple 5, which are identified as noise components. Therefore, the value of the **Grouping Effect (r)** parameter is r=3r=3, as the number of eigentriples that do not represent noise in the eigenvalue plot is 3. Although the eigentriples representing noise have been identified, there is a possibility that the number of noise components may increase. Any remaining eigentriples that are not related to trend or seasonality from the first 3 eigentriples will be grouped into the noise category.

b. Grouping Trend and Seasonal Components

After successfully grouping the noise components, the next step is to group the eigentriples associated with **trend** and **seasonality**. To identify the trend and seasonal components, 5 eigentriples are used: eigentriples 1, 2, 3, 4, and 5. A plot of the reconstructed series can be utilized to identify which eigentriples correspond to trend and seasonality. Eigentriples that exhibit smooth, slowly varying patterns in the reconstructed series are classified as trend components, while those displaying repeating, periodic patterns are classified as seasonal components. This ensures that the meaningful elements of the data are preserved in the reconstruction process

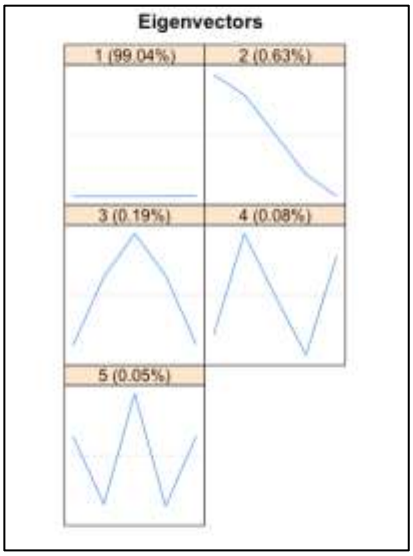


Figure 5. *Eigenvector Graph*

All components that vary slowly in the reconstructed series plot are identified as trend components. Based on Figure 5, it is evident that the series reconstructed by eigentriple 1 and eigentriple 2 contains components that vary linearly and gradually. Therefore, eigentriple 1 and eigentriple 2 are grouped into the **trend category**.

After successfully grouping the eigentriples related to trend, the next step is to group the eigentriples associated with seasonality. The grouping of seasonal eigentriples is based on the similarity of eigenvalues between two consecutive eigentriples. In the plot of the reconstructed series, an eigentriple demonstrates a seasonal pattern that is similar and has the same seasonal period as another eigentriple. For instance, eigentriples 3 and 4 exhibit seasonal components, indicating their periodic and repetitive behavior. These eigentriples are therefore grouped into the **seasonality category**.

Therefore, the remaining eigentriples, which are not included in the trend and seasonality components from the total of 5 eigentriples, will be grouped into the **noise category**.

Table 7. Eigentriples and Their Associated Components

Jenis Komponen	***Eigentriple***
Musiman	3,4
Trend	1,2
Noise	5

A. ***Diagonal Averaging***

In the final stage of the reconstruction process, **diagonal averaging** is used to reconstruct each component. Each component can be reconstructed using eigentriples 1 and 2. Figure 6 shows the results of the trend component that has been reconstructed using these two eigentriples.

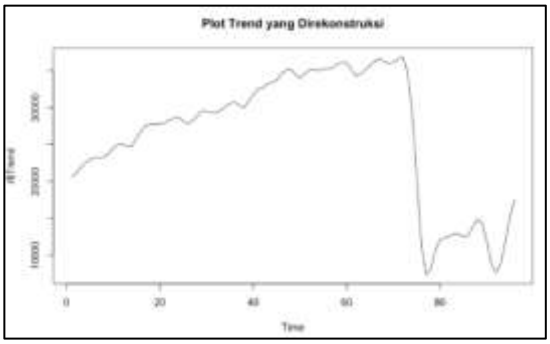


Figure 6. Reconstructed Trend Plot

In the diagonal averaging stage, the seasonal components are reconstructed using eigentriples 3 and 4. Figure 6 displays the results of the seasonal component that has been reconstructed using these two eigentriples.

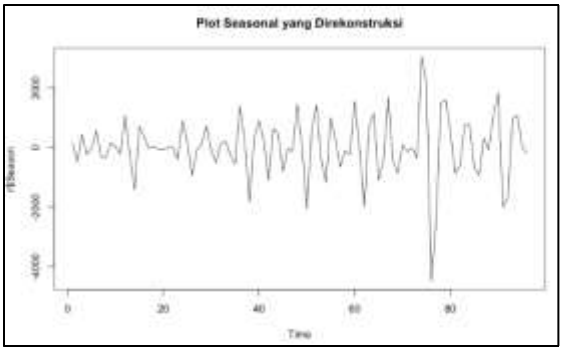


Figure 7. Reconstructed Seasonality

In the diagonal averaging step, the noise components are reconstructed using eigentriple 5. Figure 7 shows the results of the reconstruction of the noise component.

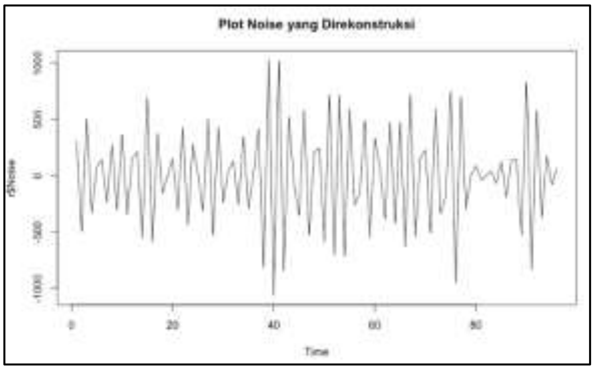


Figure 8. Plot of Reconstructed Noise

3. Accuracy of SSA Method Forecasting Error

After obtaining forecasts using the Singular Spectrum Analysis (SSA) method, the next step is to evaluate the model's performance by comparing the forecasted values with the actual values in the test dataset. In this case, the **Mean Absolute Percentage Error (MAPE)** is used as the evaluation metric.

The formula for MAPE is:

$$MAPE = \frac{1}{N}\sum_{t=1}^{N}\frac{|X_t-\hat{X}_t|}{X_t}\, x\ 100\%$$

$$MAPE = \frac{1}{24}\sum_{t=1}^{1}\frac{|29017-34556.5|}{29017}+\frac{|26259-35361.5|}{26259}+\cdots+\frac{|35059-35635.3|}{35059}\times 100\%$$

$$MAPE = \mathbf{13{,}33}\%$$

Based on the MAPE calculation above, the Singular Spectrum Analysis (SSA) method achieves a MAPE of **13.33%**. A forecast is considered good if its MAPE falls between **10% and 20%**. Therefore, a MAPE of **13.33%** indicates that the model has good forecasting capability.

Comparison of MAPE Values for HWES (Additive and Multiplicative) and SSA Methods

By comparing the MAPE values of each method, we can determine the most effective method for forecasting the number of PT KAI passengers in Indonesia. Based on the comparison:

Table 8. Comparison of MAPE Values

Metode	MAPE
HWES *Additive*	**10.75%**
HWES *Multiplicative*	14.50%
Singular Spectrum Analysis	13.33%

The **MAPE value** for forecasting the number of PT KAI passengers in Indonesia using the **Holt-Winters Exponential Smoothing Additive model** indicates that this method is the most appropriate for predicting the number of PT KAI passengers over the next 12 periods, as shown in **Table 8**:

Forecasting Using the Best Method

Based on the results of the MAPE values, the Holt Winters Exponential Smoothing method is more reliable for predicting the number of PT KAI passengers in Indonesia. In the Holt Winters Exponential Smoothing method, the model used for forecasting is additive which produces parameter values of alpha (α) of 0.4, beta (β) of 0.2 and gamma (γ) of 0.7. The following is data from forecasting the number of PT KAI passengers in Indonesia for the next 12 month period, starting from January 2024 to December 2024 at PT Kereta Api Indonesia. Based on the syntax process in the Rstudio software, the following are the forecasting results for the additive model:

Table 9. Forecasting the Number of PT KAI Passengers for the Next 12 Periods

Periode	Peramalan
Januari 2024	34788.62
Februari 2024	30552.5
Maret 2024	32510.11
April 2024	28832.11
Mei 2024	28830.26
Juni 2024	27269.49
Juli 2024	27602.94
Agustus 2024	26095.79
September 2024	27368.59
Oktober 2024	30296.65
November 2024	30836.69
Desember 2024	33518.44

Based on the forecasting results shown in table 9, it is known that in 2024, the lowest number of train passengers will occur in August with 26095.79 passengers, while the highest number of passengers will occur in January with 34788.62 passengers. Apart from that, there is also a comparison between actual data and the results of forecasting the number of train passengers in Indonesia using the Holt Winters method.

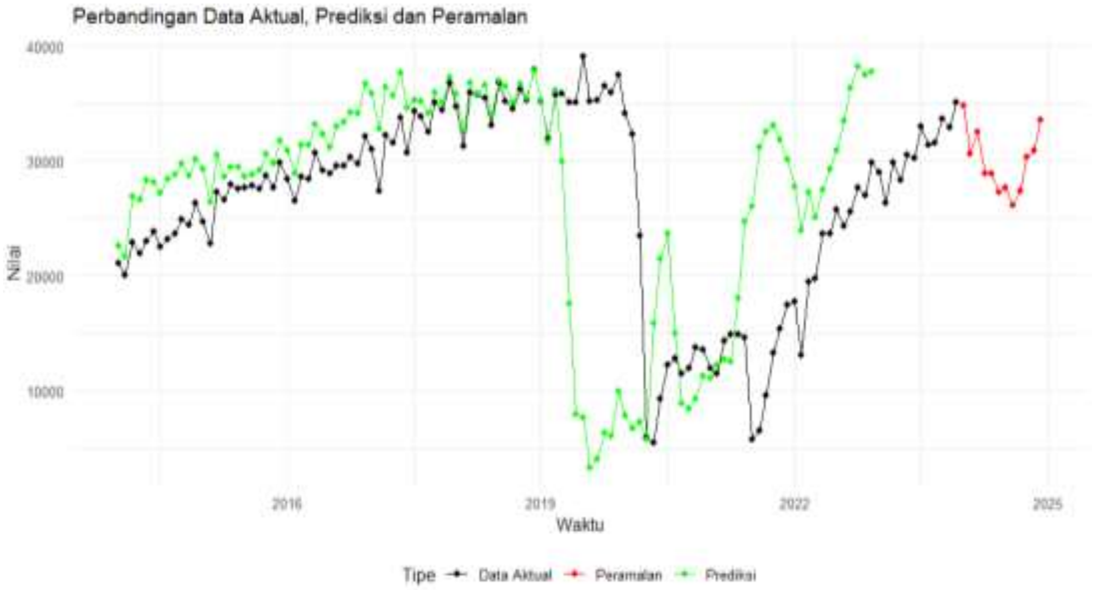


Figure 9. Forecasting the Number of PT KAI Passengers in Indonesia

Figure 9 is a graph that displays a comparison between actual data, predicted data and forecast data on the number of train passengers in Indonesia. This graph uses the Holt Winters Exponential Smoothing method with an additive model, which produces predictions that tend to follow actual data patterns. The y-axis shows the number of train passengers in Indonesia, while the x-axis shows the year. The black line on the graph represents actual data, while the green line shows predictions based on previous year's data and the red line shows the forecasting pattern. This graph illustrates that the forecasting model used is able to capture and represent patterns in the data well.

CONCLUSION

The application of the Holt Winters Exponential Smoothing (HWES) method in forecasting the number of passengers at PT Kereta Api Indonesia, using an additive model produces smoothing parameter values of alpha (α) = 0.4, beta (β) = 0.2 and gamma (γ) = 0.7. Based on the size of the error, the calculated MAPE value in this additive model is 10.75%. Meanwhile, the multiplicative model produces smoothing parameter values alpha (α) = 0.6, beta (β) = 0.1 and gamma (γ) = 0.8. Based on error accuracy, the calculated MAPE value in this multiplicative model is 14.50%. The most optimal HWES method for predicting is the additive model because the MAPE value is smaller. Based on the results of trial and error in applying the Singular Spectrum Analysis (SSA) method, in predicting the number of passengers at PT Kereta Api Indonesia it produces a window length (L) = 5 with an error accuracy in the MAPE value of 13.33%.

Comparison of MAPE values from the Holt Winters Exponential Smoothing (HWES) and Singular Spectrum Analysis (SSA) methods to predict the number of passengers for PT Kereta Api Indonesia obtained a comparison of MAPE values. So the Holt Winters Exponential Smoothing Additive method is the best method for analyzing forecasting the number of PT Kereta Api Indonesia passengers.

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