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CoVaR Modelling using QRNN Based on Quantile Regression And Quantile Autoregressive Models with Stochastic Search Variable Selection for LQ45

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Abstract

Stock price fluctuations under turbulent market conditions can increase extreme loss risk and trigger risk contagion among firms. In highly liquid stock groups such as LQ45, Value-at-Risk (VaR) is insufficient because it only measures individual risk and does not explain how a firm's risk changes when other firms are in distress. Therefore, this study employs Conditional Value-at-Risk (CoVaR) to measure extreme firm risk conditional on the distress condition of other firms. This study models the CoVaR of LQ45 firms using Quantile Regression Neural Network (QRNN), with VaR estimated dynamically through Quantile Autoregression (QAR), and applies Stochastic Search Variable Selection (SSVS) to identify the most informative inputs. The data consist of daily log returns of 24 LQ45 firms from 15 September 2020 to 30 January 2026. Candidate inputs include QAR-based VaR of other firms, IHSG return, USD/IDR return, gold return, net foreign flow, and historical volatility. VaR and CoVaR are estimated at the 5% and 1% quantiles. The results show that SSVS simplifies the input structure from 28 candidates to an average of 2.4 selected inputs, with IHSG return as the dominant systemic factor. Backtesting and pinball loss evaluation indicate that QRNN-SSVS produces a more parsimonious and efficient CoVaR model, with adequate violation calibration and comparable predictive accuracy to QRNN without SSVS.

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INTRODUCTION

The stock market is an interconnected system, particularly among highly liquid stocks such as those in Indonesia's LQ45 index. Stocks included in the LQ45 index have high trading activity, so that a change in the condition of one issuer can affect other issuers through market factors, shifts in investor sentiment, or economic conditions. The dynamic movement of stock prices exposes investors to the risk of extreme losses, especially during periods of market stress. Therefore, risk measurement must consider not only the individual risk of an asset but also the ability to capture the interconnectedness of risk among assets within a financial system. In market-risk measurement, Value-at-Risk (VaR) is one of the most widely used approaches for estimating potential losses at a given confidence level (Duffie & Pan, 1997). However, VaR has limitations because it only describes risk at the individual or portfolio level and cannot explain how the risk of an asset changes when another asset or the market system is under extreme conditions. Moreover, the characteristics of stock returns—being volatile, not always normally distributed, and exhibiting tail-risk phenomena—require methods that can better capture the dynamics of extreme risk (Cont, 2001).

To overcome these limitations, Adrian and Brunnermeier (2016) introduced Conditional Value-at-Risk (CoVaR) as a systemic-risk measure that quantifies the risk contribution of an institution or asset to the system when it is in a particular condition. CoVaR is able to capture dependence in the tail of the distribution (tail dependence), making it more relevant for describing systemic risk and risk-spillover effects. Several studies have shown that the CoVaR approach is effective in identifying institutions that contribute to financial-system risk (Wijoyo et al., 2022; Kholishoh et al., 2024). Nevertheless, CoVaR estimation still faces challenges because the relationship between returns and risk factors can be nonlinear and may vary with market conditions. Quantile-based approaches are among the methods capable of capturing such characteristics. Koenker and Bassett (1978) introduced Quantile Regression, which allows modelling across various parts of the data distribution, including the tail. Subsequently, Koenker and Xiao (2006) developed the Quantile Autoregressive (QAR) model, which can capture asymmetric dynamics and different degrees of persistence across quantiles in time-series data. The QAR approach has also been applied to dynamic risk estimation, as shown by Dzhamtyrova and Maple (2022), who demonstrated the capability of QAR in modelling risk quantiles in time-series data.

In addition to the choice of estimation method, the selection of input variables is an important factor in producing an efficient and interpretable risk model. A model with a large number of predictor variables can increase model complexity and the risk of overfitting. Stochastic Search Variable Selection (SSVS) is a Bayesian variable-selection method that can be used to identify variables with important contributions through the posterior inclusion probability (George & McCulloch, 1993). The application of SSVS in risk modelling and forecasting has also yielded promising results, for instance in GEV regression for interval forecasting [17]. In nonlinear risk modelling, Keilbar and Wang (2022) showed that the Neural Network Quantile Regression approach can improve CoVaR estimation by capturing nonlinear relationships and systemic-risk dynamics. Furthermore, Taylor (2000) showed that the Quantile Regression Neural Network (QRNN) approach can be used to estimate the characteristics of the return distribution more flexibly.

Based on prior research, the VaR, CoVaR, QAR, QRNN, and SSVS methods have demonstrated the ability to measure extreme risk and identify relevant risk factors. However, the integration of dynamic QAR-based VaR estimation, CoVaR modelling using QRNN, and input-variable optimization with SSVS on LQ45 stocks still leaves room for development. Therefore, this study aims to develop a CoVaR model with input optimization using SSVS through the QRNN approach to measure systemic risk and spillover risk among LQ45 issuers. This approach is expected to produce a risk-measurement model that is more adaptive and efficient and able to capture the nonlinear characteristics of extreme risk in the stock market.

METHOD

2.1 Returns and Log-Returns of Stocks

In market-risk analyses such as VaR and CoVaR, the variable used is generally the return, because it can represent changes in investment value as well as the characteristics of financial time series such as fat tails and volatility clustering (Cont, 2001). The arithmetic return is defined as:

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} \tag{1}$$

The log-return is defined as:

$$r_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) \tag{2}$$

with the relationship:

$$r_{i,t} = \ln(1 + R_{i,t}), R_{i,t} = e^{r_{i,t}} - 1 \tag{3}$$

This study uses the log-return as the response variable because of its additive property and its common use in financial time-series modelling. VaR at quantile level τ is expressed as the conditional quantile of the return:

$$VaR_{i,t}(\tau) = Q_{r_{i,t}}(\tau \mid \mathcal{F}_{t-1}) \tag{4}$$

2.2 LQ45 Stock Index

The LQ45 index is an Indonesia Stock Exchange index consisting of 45 stocks with high liquidity and large market capitalization (liquid stocks with high market capitalization) (Solihin et al., 2022). The selection of constituents is based on criteria such as trading activity, market capitalization, and the company's fundamental condition, with periodic evaluation and updating. In various empirical studies, LQ45 stocks are frequently used as a representation of liquid stocks for analyses of market risk, return, and volatility, including as a benchmark in studies of conventional stock-return characteristics (Hapsari et al., 2025). However, because its composition changes periodically, the determination of the research sample needs to be adjusted to the observation period so that the analysis remains consistent and unbiased by changes in index membership.

2.3 Autoregressive (AR) Model

The Autoregressive (AR) model is a time-series model that describes the relationship between the value of a variable at the current period and the historical values of that variable. The AR(p) model uses information up to lag p to capture dependence patterns in time-series data (Box et al., 2015). In general, the AR(p) model can be written as:

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t, \tag{5}$$

2.4 Quantile Regression (QR)

Quantile Regression (QR) is a regression method used to estimate the relationship between a response variable and predictor variables at a particular quantile level. Unlike mean-based regression such as OLS, QR can describe the characteristics of the data distribution at various quantile levels, including the extreme parts relevant to financial-risk analysis (Koenker & Bassett, 1978). The conditional quantile can be formulated as:

$$Q_{Y_t}(\tau \mid x_t) = x_t^T \beta(\tau), \tag{6}$$

2.5 Quantile Autoregressive (QAR)

Quantile Autoregressive (QAR) is an extension of the autoregressive model within the quantile-regression framework, used to model the conditional quantile of a time series based on historical information. Unlike the autoregressive model, which models the conditional mean, QAR can capture the dynamics in the tail of the distribution, making it suitable for financial-risk analysis, particularly Value-at-Risk (VaR) estimation (Koenker & Xiao, 2006).

In this study, the return of an issuer at time t is modelled based on its past returns up to lag p. The QAR(p) model at quantile level τ is formulated as:

$$Q_{Y_{j,t}}(\tau \mid \mathcal{F}_{t-1}) = \phi_{0,j}(\tau) + \sum_{i=1}^{p} \phi_{i,j}(\tau) y_{j,t-i} \tag{7}$$

The QAR parameters are estimated using the quantile-regression principle by minimizing the pinball loss function. The estimation yields dynamic conditional quantiles that serve as the basis for computing VaR:

$$\widehat{VaR}_{j,t}^{QAR}(\tau) = \hat{Q}_{Y_{j,t}}(\tau|\mathcal{F}_{t-1}) = \hat{\phi}_j(\tau)^T \boldsymbol{x}_{j,t-1}^{QAR} \quad (8)$$

The QAR-based VaR estimate is used to describe the individual risk of an issuer and to determine the distress condition that forms the basis for constructing the CoVaR model.

2.6 Value-at-Risk (VaR)

Value-at-Risk (VaR) is a risk measure used to estimate the potential loss of an asset or portfolio at a given probability level. Within the return framework, VaR is defined as the lower quantile of the return distribution, so that it can describe the extreme-loss limit in the left tail of the distribution (Duffie & Pan, 1997). Let Y(j,t) denote the random return of target issuer j at time t and F(t-1) denote the information set available up to time t-1. VaR at quantile level τ is defined as:

$$VaR_{j,t}(\tau) = Q_{Y_{j,t}}(\tau|\mathcal{F}_{t-1}) \quad (9)$$

where Q(Y(j,t))(τ|F(t-1)) is the conditional quantile of issuer j's return at quantile level τ. For low quantile levels, the VaR value is generally negative because it represents an extreme-loss condition.

VaR is not computed as a static risk measure but is estimated dynamically using the QAR model based on historical return information. The QAR-based VaR estimate is used to describe the individual risk of an issuer and serves as the basis for determining the distress condition in Conditional Value-at-Risk (CoVaR) modelling. Validation of the VaR estimate is based on the violation indicator, which shows whether the actual return falls below the predicted VaR value:

$$I_{j,t}(\tau) = \mathbb{I}(y_{j,t} < \widehat{VaR}_{j,t}^{QAR}(\tau)) \quad (10)$$

The indicator value is subsequently used in the backtesting stage to evaluate the consistency of the VaR model with the targeted quantile level.

2.7 Conditional Value-at-Risk (CoVaR)

Conditional Value-at-Risk (CoVaR) is an extension of VaR used to measure the risk of one issuer while taking into account extreme conditions in another issuer. Unlike VaR, which only measures individual risk, CoVaR can describe risk interconnectedness and the spillover-risk effect among issuers when one issuer is in a distress condition (Adrian & Brunnermeier, 2016). The distress condition of conditioning issuer j* is determined based on the QAR-based VaR estimate:

$$VaR_{j^*,t}^{QAR}(\tau) = Q_{Y_{j^*,t}}(\tau|\mathcal{F}_{t-1}) \quad (11)$$

The CoVaR of target issuer j is then defined as the conditional quantile of issuer j's return when conditioning issuer j* is in a distress condition, namely:

$$P(Y_{j,t} \leq CoVaR_{j|j^*,t}^{D}(\tau)|Y_{j^*,t} \leq VaR_{j^*,t}^{QAR}(\tau), \mathcal{F}_{t-1}) = \tau \quad (12)$$

In this study, the CoVaR model uses inputs consisting of the conditioning issuer's VaR as a representation of the distress condition together with external risk factors. The initial input vector of the model can be written as:

$$z_{j|j^*,t}(\tau) = (\widehat{VaR}_{j^*,t}^{QAR}(\tau), x_{1,t-1}, x_{2,t-1}, \dots, x_{K,t-1})^T \quad (13)$$

where x(k,t-1) is a risk-factor variable available up to the previous period. The nonlinear relationship between the target issuer's return and the risk factors is modelled using a Quantile Regression Neural Network (QRNN), so that the CoVaR estimate is formulated as:

$$\widehat{CoVaR}_{j|j^*,t}^{QRNN}(\tau) = f_\tau(z_{j|j^*,t}(\tau); \hat{\theta}_\tau) \quad (14)$$

2.8 Nonlinearity Test

A nonlinearity test is conducted to determine whether the relationship between the target issuer's return and the CoVaR model inputs is linear or nonlinear. This test is important because QRNN is used to capture complex quantile relationships under extreme conditions. This study uses the neural-network-based Teräsvirta test to detect the presence of nonlinear patterns in the regression relationship (Teräsvirta et al., 1993). The initial linear model is expressed as:

$$y_{j,t} = \beta_0 + \beta^T z_{j|j^*,t}(\tau) + \varepsilon_{j,t} \quad (15)$$

Next, the test is performed by adding a nonlinear component:

$$\hat{\varepsilon}_{j,t} = C_\tau + \boldsymbol{\psi}_\tau^T \boldsymbol{z}_{j|j^*,t}(\tau) + \boldsymbol{\delta}_\tau^T \boldsymbol{G}\left(\boldsymbol{z}_{j|j^*,t}(\tau)\right) + u_{j,t} \tag{16}$$

The test hypotheses are H0: δ=0, indicating no nonlinear relationship, and H1: δ≠0, indicating the presence of a nonlinear component. Rejection of H0 provides the basis that a nonlinear model such as QRNN is more appropriate for CoVaR estimation.

2.9 Quantile Regression Neural Network (QRNN)

The Quantile Regression Neural Network (QRNN) is an extension of quantile regression that uses a neural-network structure to estimate conditional quantiles nonlinearly. This approach is used because the relationship among the target issuer's return, the conditioning issuer's distress condition, and market risk factors can be nonlinear. In this study, QRNN is used to estimate the conditional quantile of returns as the basis for constructing CoVaR.

QRNN uses a single-hidden-layer feedforward neural-network architecture, with inputs consisting of the conditioning issuer's VaR and exogenous risk covariates, and an output consisting of the conditional quantile of the target issuer's return. The QRNN conditional-quantile function is expressed as:

$$Q_{Y_{j,t}}(\tau \mid \mathbf{z}_{j|j^*,t}) = f_\tau(\mathbf{z}_{j|j^*,t}; \theta_\tau) \tag{17}$$

where f(τ)(·) is the nonlinear QRNN function and θ(τ) is the network's parameter set. The nonlinear relationship in the hidden layer is formed using the tanh activation function:

$$g(a) = \tanh(a) = \frac{e^a - e^{-a}}{e^a + e^{-a}} \tag{18}$$

while the output layer uses a linear function because the CoVaR value can be negative. The general form of QRNN with one hidden layer is:

$$f_\tau(\mathbf{z}_{j|j^*,t}; \theta_\tau) = c^{(\tau)} + \sum_{\ell=1}^{L} v_\ell^{(\tau)} h_{\ell,t}^{(\tau)} \tag{19}$$

The CoVaR estimate is obtained from the QRNN conditional-quantile output:

$$\widehat{CoVaR}_{j|j^*,t}^{QRNN}(\tau) = f_\tau(\mathbf{z}_{j|j^*,t}; \hat{\theta}_\tau) \tag{20}$$

The QRNN network parameters are estimated by minimizing the pinball loss (quantile loss):

$$\rho_\tau(u) = u(\tau - I(u < 0)) \tag{21}$$

2.10 Stochastic Search Variable Selection (SSVS)

Stochastic Search Variable Selection (SSVS) is a Bayesian variable-selection method that uses an inclusion indicator and a spike-and-slab prior (George & McCulloch, 1993). In this study, SSVS is used as embedded variable selection within QRNN to select the covariates that contribute to CoVaR estimation. Each exogenous covariate is governed by an inclusion indicator γ(k):

$$\gamma_k = \begin{cases} 1, & \text{kovariat aktif dalam model} \\ 0, & \text{kovariat tidak aktif dalam model} \end{cases} \tag{22}$$

so that the QRNN input vector becomes:

$$\tilde{\mathrm{z}}_{j|j^*,t}^{(\tau)} = (\widehat{VaR}_{j^*,t}^{QAR}(\tau), \gamma_1 x_{1,t-1}, \dots, \gamma_K x_{K,t-1})^T \tag{23}$$

The inclusion indicator determines the prior of the network weights through a spike-and-slab distribution:

$$\omega_k^{(\tau)} \mid \gamma_k \sim (1 - \gamma_k) N(0, c_{0,k}^2 I_L) + \gamma_k N(0, c_{1,k}^2 I_L) \tag{24}$$

where the spike component shrinks the weights of irrelevant covariates, while the slab provides flexibility for contributing covariates. Covariate selection is based on the Posterior Inclusion Probability (PIP):

$$PIP_k = P(\gamma_k = 1 \mid data) \tag{25}$$

2.11 Backtesting and Quantitative Accuracy Evaluation

Backtesting is used to evaluate the performance of the quantile models by comparing the VaR and CoVaR estimates against the actual returns in the testing period. The evaluation uses the Kupiec Proportion of Failures (POF) Test and the Pinball Loss. The Kupiec POF Test examines the consistency of the violation proportion with the targeted quantile level, while the pinball loss measures the accuracy of the quantile prediction. The proportion of shortfall is computed as:

$$\widehat{PoS}_\tau^{CoVaR} = \frac{1}{n_{test}} \sum_{t \in \mathcal{T}_{test}} I(y_{j,t} < \widehat{CoVaR}_{j|j^*,t}(\tau)) \tag{26}$$

The Kupiec POF Test is used to examine whether the violation proportion is consistent with the targeted quantile level. The test statistic is expressed as:

$$LR_{POF} = -2\ln\left[\frac{(1-\tau)^{n_{test}-N_\tau}\tau^{N_\tau}}{(1-\hat{\pi}_\tau)^{n_{test}-N_\tau}\hat{\pi}_\tau^{N_\tau}}\right] \quad (27)$$

where $\hat{\pi}$ is the violation proportion. A model is considered to have good quantile coverage if the violation proportion is close to the quantile level τ. In addition, the accuracy of the quantile prediction is evaluated using the pinball loss:

$$PL_\tau = \frac{1}{n_{test}}\sum_{t\in\mathcal{T}_{test}}\rho_\tau\,(y_{j,t} - \hat{q}_{j,t}(\tau)) \quad (28)$$

A smaller pinball loss indicates a more accurate quantile prediction. Both evaluations are used to compare the performance of the CoVaR-QRNN model with and without SSVS.

2.12 Data Source

This study uses secondary daily time-series data from issuers that were consistently part of the LQ45 index during the period September 2020–January 2026. The data were obtained from the Indonesia Stock Exchange (IDX), Yahoo Finance, Investing.com, Bank Indonesia, and the Financial Services Authority (OJK), aligned according to the IDX trading calendar so that all observations represent the same market time. The research variables consist of a response variable, namely the daily log-return of LQ45 issuers, and explanatory variables comprising the conditioning issuers' QAR-based VaR and macro-financial covariates (the IHSG composite index, net foreign flow, historical volatility, USD/IDR, and the gold price), constructed in lagged form to avoid look-ahead bias.

2.13 Data Analysis

In general, the analysis consists of the following steps:

1. Data Preprocessing: aligning the data according to the IDX calendar, handling missing values, computing log-returns, constructing macro-financial covariates, and transforming all variables into lagged form. A stationarity test (ADF) is then performed, and the data are split into in-sample and out-of-sample sets.
2. VaR Estimation (QAR Model): estimating the conditioning issuers' VaR using Quantile Autoregression (QAR) to obtain a distress-condition measure to be used as input for the CoVaR model.
3. CoVaR Estimation (QRNN Model): building a QRNN-based CoVaR model to capture the nonlinear relationship among the target issuer's return, the conditioning issuer's VaR, and exogenous covariates by minimizing the pinball loss.
4. SSVS (Variable Selection): integrating Stochastic Search Variable Selection (SSVS) into the QRNN using a spike-and-slab prior and Gibbs sampling to produce the Posterior Inclusion Probability (PIP) and select relevant covariates.
5. Model Evaluation: evaluating model performance on out-of-sample data using the Kupiec POF Test for quantile coverage and the pinball loss for prediction accuracy, and comparing the QRNN model with and without SSVS.

RESULTS AND DISCUSSION

The initial stage of the study was carried out by estimating the distress component in the form of Value-at-Risk (VaR) using the Quantile Autoregression (QAR) approach. The estimation was performed on the daily log-returns of the 24 LQ45 issuers that were consistent throughout the observation period, at quantile levels τ=5% and τ=1%. The VaR-QAR values were subsequently used as a representation of the conditioning issuer's distress condition in constructing the QRNN-based CoVaR model.

3.1 Characteristics of the Log-Return Data

This study uses daily stock log-return data as the object of analysis. The log-return value is computed from each stock's closing price and represents the rate of return (gain or loss) received by investors. In general, all issuers have an average daily log-return close to 0, so that in terms of average performance these stocks were relatively flat over the observation period. The largest fluctuations were experienced by issuers in the commodity and energy sectors, such as MDKA (3.35%), ANTM

(3.19%), and INCO (2.92%). The most extreme loss values (daily returns) were experienced by ADRO (28.29%), INKP (17.07%), and UNVR (16.74%). The distribution pattern of the returns is presented through the boxplot in Figure 1.

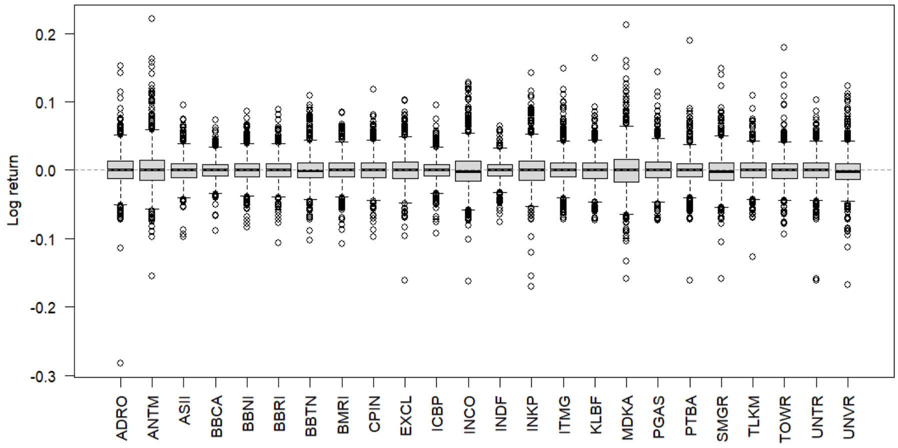

Figure 1. Boxplot of LQ45 Stock Log-Returns

Based on figure 1, the boxplot shows several extreme values in the stock returns, which supports the use of a quantile-based approach in risk modelling. Before time-series modelling, a stationarity test was performed using the Augmented Dickey-Fuller (ADF) test. The results show that all log-return variables have a p-value below 0.010, so that the data can be concluded to be stationary and to satisfy the assumptions for QAR-based modelling.

3.2 QAR-Based VaR Estimation as the Distress Component

a. QAR Model for Each Issuer (τ=5%)

Based on table 2, VaR modelling was performed using Quantile Autoregression (QAR) on the daily log-returns at quantile level τ=5% to capture extreme-loss risk through the dynamics of past returns. The results show that each issuer has a different lag structure according to the characteristics of its return movement.

Table 2. Example QAR Models for Issuers at τ=5%

Issuer	Model	Equation
ADRO	QAR(1)	$VaR_{j,t}^{QAR}(\tau) = \hat{\phi}_{0,j}(\tau) + \hat{\phi}_{1,j}(\tau)y_{j,t-1}$ $= -0.03852(0.05) + 0.01228(0.05)y_{j,t-1}$
TLKM	QAR(2)	$VaR_{j,t}^{QAR}(\tau) = \hat{\phi}_{0,j}(\tau) + \hat{\phi}_{1,j}(\tau)y_{j,t-1} + \hat{\phi}_{2,j}(\tau)y_{j,t-2}$ $= -0.02547(0.05) - 0.00711(0.05)y_{j,t-1} + 0.00401(0.05)y_{j,t-2}$
MDKA	QAR(1)	$VaR_{j,t}^{QAR}(\tau) = \hat{\phi}_{0,j}(\tau) + \hat{\phi}_{1,j}(\tau)y_{j,t-1}$ $= -0.04548(0.05) - 0.08407(0.05)y_{j,t-1}$
ANTM	QAR(1)	$VaR_{j,t}^{QAR}(\tau) = \hat{\phi}_{0,j}(\tau) + \hat{\phi}_{1,j}(\tau)y_{j,t-1}$ $= -0.04206(0.05) + 0.05435(0.05)y_{j,t-1}$
INCO	QAR(1)	$VaR_{j,t}^{QAR}(\tau) = \hat{\phi}_{0,j}(\tau) + \hat{\phi}_{1,j}(\tau)y_{j,t-1}$ $= -0.03953(0.05) - 0.02673(0.05)y_{j,t-1}$

After modelling the log-return data in R, the QAR-based VaR values were obtained. The VaR graphs for issuers with extreme conditions (such as MDKA and ANTM) and stable conditions (such as TLKM and ADRO) at quantile level τ=5% are shown in Figure 2.

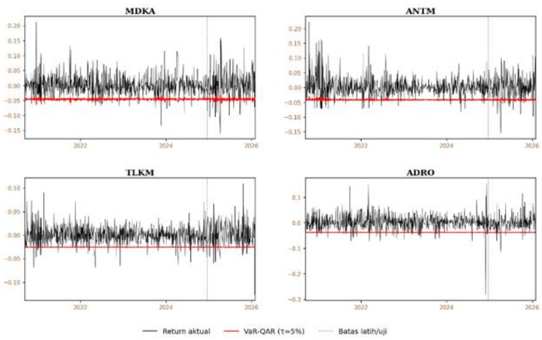

Figure 2. Comparison of actual returns and VaR-QAR estimates (τ=5%)

Figure 2 presents the comparison between actual returns and the VaR estimated using the QAR basis at quantile level τ=5% for each issuer. The black line represents the actual return, while the red line shows the VaR-QAR value, which describes the maximum loss limit at the 95% confidence level. In Figure 2, TLKM and ADRO show relatively stable return patterns with smaller fluctuations around the VaR value. By contrast, MDKA and ANTM—two issuers in the mining sector—exhibit much larger return fluctuations with high volatility, especially in certain periods. These differing patterns indicate that the VaR-QAR model is sensitive to each issuer's volatility and can distinguish between stable and extreme conditions across issuers.

Table 3. Summary of VaR-QAR Estimates (τ=5%)

Issuer	Mean VaR (%)	SD VaR (%)
ADRO	-3.851	0.033
TLKM	-2.547	0.016
MDKA	-4.552	0.282
ANTM	-4.198	0.173
INCO	-3.952	0.078

Based on the estimation results in table 3, at the 95% confidence level the VaR-QAR (τ=5%) estimates indicate that each issuer has a maximum daily potential loss corresponding to its respective VaR value. Assuming an investor invests IDR 100 million, the possible daily loss under extreme market conditions (the worst 5% of cases) is as follows: MDKA may incur a loss of 4.55% (IDR 4.55 million), ANTM 4.19% (IDR 4.19 million), while TLKM is 2.54% (IDR 2.54 million) and ADRO is 3.85% (IDR 3.85 million).

b. QAR Model for Each Issuer (τ=1%)

QAR modelling was also performed at quantile level τ=1% to capture extreme-loss conditions at the 99% confidence level. The lag order was determined based on the quantile-regression estimation results, so that each issuer has a different model specification according to its return characteristics.

Table 4. Example QAR Models for Issuers at τ=1%

Issuer	Model	Equation
ANTM	QAR(1)	$VaR_{j,t}^{QAR}(\tau) = \hat{\phi}_{0,j}(\tau) + \hat{\phi}_{1,j}(\tau)y_{j,t-1}$ $= -0.07025(0.01) - 0.00947(0.01)y_{j,t-1}$
CPIN	QAR(1)	$VaR_{j,t}^{QAR}(\tau) = \hat{\phi}_{0,j}(\tau) + \hat{\phi}_{1,j}(\tau)y_{j,t-1}$ $= -0.04884(0.01) + 0.01514(0.01)y_{j,t-1}$
MDKA	QAR(1)	$VaR_{j,t}^{QAR}(\tau) = \hat{\phi}_{0,j}(\tau) + \hat{\phi}_{1,j}(\tau)y_{j,t-1}$ $= -0.06863(0.01) + 0.02491(0.01)y_{j,t-1}$

SMGR	QAR(1)	$VaR_{j,t}^{QAR}(\tau) = \hat{\phi}_{0,j}(\tau) + \hat{\phi}_{1,j}(\tau)y_{j,t-1}$ $= -0.05382(0.01) + 0.03158(0.01)y_{j,t-1}$
INCO	QAR(1)	$VaR_{j,t}^{QAR}(\tau) = \hat{\phi}_{0,j}(\tau) + \hat{\phi}_{1,j}(\tau)y_{j,t-1}$ $= -0.06458(0.01) + 0.05435(0.01)y_{j,t-1}$

After modelling the log-return data in R, the QAR-based VaR values were obtained. The VaR graphs for issuers with extreme conditions (such as ANTM, MDKA, and INCO) and a stable condition (such as CPIN) at quantile level τ=1% are shown in Figure 3.

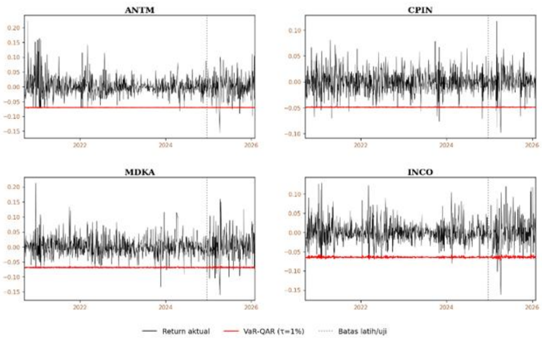

Figure 3. Comparison of Actual Returns and VaR-QAR (τ=1%)

Figure 3 presents the comparison between actual returns and the VaR estimated using the QAR basis at quantile level τ=1% for each issuer. Based on the graphs, CPIN shows a relatively stable return pattern with smaller fluctuations around the VaR value. By contrast, ANTM, MDKA, and INCO—issuers in the mineral and precious-metal mining sector—exhibit much larger return fluctuations with high volatility, especially in certain periods.

Table 5. Summary of VaR-QAR Estimates (τ=1%)

Issuer	Mean VaR (%)	SD VaR (%)
ANTM	-7.02	0.0003
CPIN	-4.88	0.00032
MDKA	-6.86	0.00084
INCO	-6.45	0.00158
SMGR	-5.38	0.00077

Based on Table 5, at the 99% confidence level the VaR-QAR (τ=1%) estimates indicate that each issuer has a maximum daily potential loss corresponding to its respective VaR value. Assuming an investor invests IDR 100 million, the possible daily loss under extreme market conditions (the worst 1% of cases) is as follows: ANTM may incur a loss of 7.02% (IDR 7.02 million), MDKA 6.86% (IDR 6.86 million), while INCO is 6.45% (IDR 6.45 million) and CPIN is 4.88% (IDR 4.88 million).

3.3 Input-Vector Construction and Multicollinearity Diagnostics

The CoVaR model input vector was constructed using a pooled approach consisting of the conditioning issuers' VaR-QAR components, macro-financial covariates, and the target issuer's historical volatility (HV20). All variables use information from period t-1 to avoid look-ahead bias. Before QRNN modelling and SSVS selection, a multicollinearity check was performed on the input variables.

Table 6. Multicollinearity Diagnostic Results

τ	Mean Correlation	Max Correlation	Pairs |r|>0.90	CN
5%	0.172	0.661	0	6.01
1%	0.176	0.661	0	6.12

Based on Table 6, the input vector is in good condition. The mean correlation among variables is low, around 0.17 (0.172 at τ=5% and 0.176 at τ=1%), with a maximum correlation of only 0.66, so that there is no pair of variables with an absolute correlation above 0.90. The condition number (CN) is 6.01 at τ=5% and 6.12 at τ=1%, still far below the common threshold that indicates a multicollinearity problem, namely >30. This indicates that each issuer's VaR-QAR carries relatively distinct information because it is driven by the lagged returns of its respective issuer, rather than by a single common market factor. Accordingly, the inputs used in the QRNN-based CoVaR model are appropriate, and the SSVS selection process is not disturbed by multicollinearity.

3.4 Nonlinearity Test

Before building the QRNN model, a Teräsvirta nonlinearity test was performed to characterize the relationship between the target issuer's return and the input variables.

Table 7. Summary of Teräsvirta Nonlinearity Test Results

Quantile	Nonlinear	Linear
$\tau = 5\%$	21	3
$\tau = 1\%$	20	4
Total	41	7

Based on Table 7, of the 48 issuer–quantile combinations (24 issuers × 2 quantiles), 41 combinations (>85%) reject linearity, with a balanced distribution across quantiles (21 at τ=5% and 20 at τ=1%). The prevalence of these nonlinear relationships provides the empirical basis for using the QRNN model.

3.5 Estimation of the Baseline CoVaR-QRNN Model and Its Benchmark

The model was estimated for the 24 target issuers at 2 quantile levels (48 models in total). For each issuer–quantile pair, the 28-dimensional input vector contains the VaR-QAR of the other 23 issuers as a representation of cross-issuer distress and five exogenous covariates, paired with the target issuer's log-return as the response variable. The data were split chronologically into the first 884 days as training data and the last 222 days as testing data. The number of hidden neurons was chosen based on the validation pinball loss among the candidates L = 1, 2, 3, 4, and 5. The final model was then re-estimated on the entire training set using the best number of neurons.

The pinball values on the testing data are generally higher than on the training data, on average about 1.8 times higher at τ=5% and 3 times higher at τ=1%, with an increase for almost all issuers (22 of 24 issuers at τ=5% and all issuers at τ=1%). The 2025 testing period was indeed marked by more turbulent market conditions than the 2020–2024 training period, so that the model faced market conditions that were more difficult to predict. The testing pinball loss at τ=1% is generally smaller than at τ=5%. This occurs because the pinball-loss penalty depends on the quantile level used. At τ=5%, the largest pinball loss is found for ADRO (0.002573) and the smallest for INDF (0.001242). At τ=1%, the largest pinball loss is also found for ADRO (0.000613) and the smallest for BBCA (0.000291).

3.6 Posterior Inclusion Probability and Drivers of Systemic Risk

After modelling QRNN-based CoVaR with SSVS using R software, the estimation results are presented on table 8 and table 9.

a. CoVaR Model Based on QRNN with SSVS (τ=5%)

Table 8. CoVaR Model Based on QRNN with SSVS (τ=5%)

No.	Issuer	M^*	L^*	Selected Inputs
1.	ANTM	1	1	GOLD(0.82)
2.	BBCA	3	2	VaR BBNI(0.49); IHSG(0.33); VaR INKP(0.13)
3.	INCO	1	3	GOLD(0.94)
4.	INDF	3	1	VaR SMGR(0.08); VaR MDKA(0.08); VaR UNVR(0.07)
5.	MDKA	1	5	GOLD(0.97)

Table 8 presents the selected inputs together with their PIP values for each issuer, along with the number of selected inputs (M*) and the selection status. The emerging pattern is highly interpretable in economic terms. The strongest driver is the gold return, which becomes the single input for the three metal-mining issuers with very high PIPs: MDKA (0.97), INCO (0.94), and ANTM (0.82), reflecting the direct link between these issuers and metal-commodity prices. The next strongest driver is the IHSG return for ICBP (PIP 0.93); the IHSG is in fact the most frequently selected input overall (appearing for nine issuers at τ=5%), yet for most issuers it enters only as a low-PIP background factor and becomes a dominant driver only for ICBP. Beyond market and commodity factors, strong spillover effects among issuers are evident: BMRI is driven by VaR UNTR (0.80) and BBRI by VaR MDKA (0.60), indicating cross-sectoral risk transmission between banking and the heavy-equipment and mining sectors. Most other issuers fall under the fallback status (M*=3) with diffuse, low PIPs, meaning that no single factor dominates their systemic risk. The QRNN-based CoVaR-SSVS values at τ=5% are visualized in Figure 4.

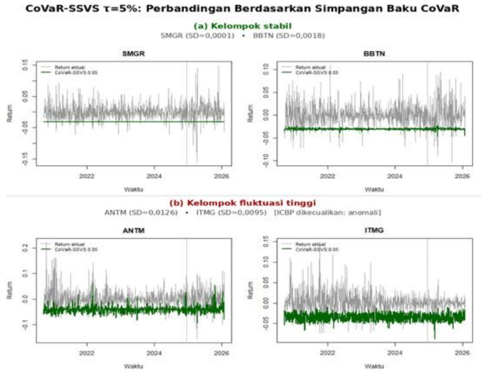

Figure 4. CoVaR-QRNN-SSVS Values at Quantile τ=5%

Figure 4 presents the QRNN-based CoVaR-SSVS values at quantile level τ=5%. Issuers whose systemic risk is governed by a single dominant input tend to display a smoother, more stable CoVaR path, whereas issuers selected through the fallback mechanism, which combine several inputs, exhibit a more responsive path that reacts to changing market conditions, especially during periods of systemic stress.

b. CoVaR Model Based on QRNN with SSVS ($\tau = 1\%$)

Table 9. CoVaR Model Based on QRNN with SSVS (τ=1%)

No.	Issuer	M^*	L^*	Selected Inputs
1.	ANTM	1	2	GOLD(0.86)
2.	BBCA	1	1	VaR BBNI(0.65)
3.	INCO	1	3	GOLD(1.00)
4.	INDF	3	1	VaR MDKA(0.13); VaR SMGR(0.11); Kurs(0.07)
5.	MDKA	1	2	GOLD(0.97)

After modelling QRNN-based CoVaR with SSVS based on table 9, the estimation results at quantile level τ=1% are visualized in Figure 5.

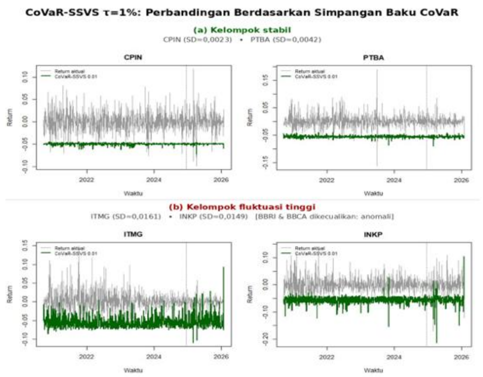

Figure 5. CoVaR-QRNN-SSVS Values at Quantile τ=1%

Figure 5 presents the QRNN-based CoVaR-SSVS values at quantile level τ=1%. As at τ=5%, single-input issuers display a relatively stable tail-risk path, while issuers combining several inputs are more responsive to changing market conditions, with the differences becoming sharper in the extreme tail.

3.7 Selected Inputs and Model Parsimony

Of the 48 issuer × quantile combinations, 13 are PIP-driven (having at least one input with PIP>0.5) and the remaining 35 go through the fallback mechanism. On average, only 2.46 inputs are selected out of the 28 available inputs, meaning that about 91% of the inputs are pruned. The selection pattern is dichotomous: PIP-driven models reduce to a single dominant input (M*=1), whereas fallback models take the three highest-PIP inputs (M*=3). This degree of pruning confirms that most inputs in the pooled vector do not provide informative contributions for each issuer specifically. After the selection procedure was established, the next stage presents the backtesting results for each model to assess whether VaR-QAR, CoVaR-QRNN, and CoVaR-QRNN-SSVS produce adequate risk estimates in the testing period. The evaluation uses the Kupiec test (coverage), the Christoffersen test (independence of violations), the violation rate π, and the pinball loss; a model is deemed satisfactory when the test p-value exceeds 0.05. A summary is presented in Table 10.

Table 10. Summary of Backtesting per Model and Quantile (Satisfied if p-value > 0.05)

Model	Kupiec Satisfied	Christoffersen Satisfied	Mean π	Mean pinball
		$\tau = 5\%$		
VaR-QAR	9/24	21/24	0.0811	0.003134
CoVaR-QRNN	8/24	21/24	0.0980	0.003443
CoVaR-QRNN-SSVS	8/24	23/24	0.0880	0.003348
		$\tau = 1\%$		
VaR-QAR	12/24	23/24	0.0253	0.001161
CoVaR-QRNN	2/24	22/24	0.0520	0.001427
CoVaR-QRNN-SSVS	8/24	23/24	0.0349	0.001312

Based on Table 10, among the three models VaR-QAR is the best calibrated (Kupiec satisfied for 9/24 issuers at τ=5% and 12/24 at τ=1%). This is rational because VaR-QAR models the return quantile of each issuer directly and is therefore easier to calibrate, whereas CoVaR is a conditional (systemic) measure that is inherently harder to calibrate. At the issuer level, several names such as ADRO, CPIN, PGAS, and UNTR consistently satisfy both the Kupiec and Christoffersen tests at both quantiles, whereas highly volatile issuers such as BBCA, BMRI, UNVR, TOWR, MDKA, and KLBF fail in almost all tests. The highest violation count occurs at τ=1%; under the unselected CoVaR-QRNN model, UNVR records 27 violations ($\pi \approx 0.122$) against an expectation of about two.

3.8 Performance Comparison of CoVaR-QRNN With and Without SSVS

A direct comparison between CoVaR-QRNN with and without SSVS shows a consistent advantage for the selected model. In terms of coverage, the clearest difference appears at the extreme quantile τ=1%: CoVaR-QRNN-SSVS satisfies the Kupiec test for 8 of 24 issuers, whereas the full model does so for only 2 of 24. At this quantile SSVS even dominates outright—it improves six issuers that previously failed (ADRO, ANTM, BBTN, ITMG, PGAS, and UNTR) without making any issuer worse. Its mean violation rate is also closer to the target (π = 0.0349 at τ=1% and 0.0880 at τ=5%, versus 0.0520 and 0.0980 for the full model), and the worst violation count drops from 27 (UNVR) to a maximum of 17 (KLBF).

In terms of independence, SSVS is superior on the Christoffersen test (23/24 at both quantiles, versus 21/24 and 22/24). In terms of accuracy, the SSVS pinball loss is smaller for 32 of the 48 combinations (14/24 at τ=5% and 18/24 at τ=1%), with a lower mean pinball at both quantiles. Overall, these results confirm that pruning about 91% of the inputs through SSVS not only makes the model far more parsimonious and interpretable, but actually improves the adequacy and accuracy of the CoVaR estimates relative to the full model.

CONCLUSION

This study develops a QRNN-based CoVaR model with input optimization through SSVS to measure systemic risk and risk spillover among LQ45 issuers. Based on the analysis of 24 issuers at quantile levels $\tau=5\%$ and $\tau=1\%$, several conclusions are obtained. First, the individual risk of each issuer is modelled with VaR-QAR whose lag order is selected through the BIC criterion; all resulting VaR values are negative, consistent with their role as extreme-loss thresholds, with the deepest potential losses at $\tau=1\%$ borne by mining issuers such as ANTM (7.03%), MDKA (6.86%), and INCO (6.46%). Second, the Teräsvirta nonlinearity test rejects a linear relationship in 41 of the 48 issuer–quantile combinations, providing the empirical basis for using QRNN; compared with linear quantile regression, CoVaR-QRNN yields better accuracy (pinball) in 30 of the 48 combinations, with an asymmetric advantage that is negligible at $\tau=5\%$ but pronounced at the extreme quantile $\tau=1\%$. Third, input selection through SSVS produces a highly parsimonious model, pruning about 91% of the inputs (on average only 2.46 of 28 inputs are selected); the identified risk drivers are economically interpretable, with the gold return as the single input for the metal-mining issuers (MDKA, INCO, and ANTM), the IHSG return as the most frequently selected aggregate-market factor and the dominant driver for ICBP, and risk spillover reflected in the selection of other issuers' VaR—such as BMRI driven by VaR UNTR and BBCA by VaR BBNI under the most extreme conditions—while VaR MDKA emerges as one of the most frequently selected inputs across issuers, marking it as a risk-transmission hub within the LQ45 network. Fourth, the backtesting results show that VaR-QAR is the best-calibrated model, while among the CoVaR models, CoVaR-QRNN-SSVS consistently outperforms the full model without selection—most clearly at $\tau=1\%$ (Kupiec satisfied for 8 of 24 issuers versus 2 of 24)—with a violation rate closer to the target and a smaller pinball loss for 32 of the 48 combinations. Thus, integrating SSVS into CoVaR-QRNN yields a model that is not only more parsimonious and interpretable, but also more adequate and accurate in measuring systemic risk among LQ45 issuers.

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